LECTURE NOTES 13

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1. Dealer Markets Models

1.1. **Glosten and Milgrom (1985) sequential model.** Assume a market place with a quote-driven protocol. That is, with competitive market makers posting bid ask spreads on one single security and trading with two types of agents: 1) insiders (informed traders), and liquidity traders (uninformed traders). The insiders receive a full informative private signal about the security's value prior to trading. All market makers are assumed to be risk-neutral and the order flow arrives sequentally at the market place (one agent's order at a time). The risk-neutrality assumption allow us to abstract from inventory considerations.

In Glosten and Milgrom (1985) the size of the order is assumed to be equal to one and agents only trade once. This is a technical point to avoid the possibility of an infinite size order from the insider. Nature chooses the unconditional future value of the security $\tilde{V}$, which can be high $\bar{V}$ with probability θ or low $\underline{V}$ with probability $1-\theta$. The standard approach in this literature after Glosten and Milgron (1985) is to set $\theta = \frac{1}{2}$ (equal probability of observing a buy or sell order).

Market makers face the informed trader with probability α and the uninformed trader with probability $1-\alpha$. The insider's strategy is to maximize profit i.e., buy when the ask price $a_t \leq \tilde{V} = \bar{V} = E\left[\tilde{V}\right] + E$ and sell when $b_t \geq \tilde{V} = \underline{V} = E\left[\tilde{V}\right] - E$. Note that E is some measure of the insider's informational advantage (or equivalently the dealer's mistake when pricing the security). Since the insider only trades once and prices are pre-set by market makers, she does not care how her trade affects the future path of the price.

At time t, dealers compete on the price of the security realizing an expected profit equal to $\prod_t = 0$. This is a Bertand type competitive setting with market makers competing in prices until profits go to zero. That is, the price equals their conditional belief about the future value of the security given the information conveyed by the direction of the trade Φ_t i.e., the signed order flow.

If the observed incoming trade is a buy order B_t, then market makers offer an ask price a_t; conversely, if they observe an incoming sell order S_t, then market makers will offer a bid price b_t. The opportunity cost that market makers face when selling the security is,

$$E\left(\prod\nolimits_t^A |\Phi_t = B_t\right) = E\left[\left(a_t - \tilde{V}\right) = E\,|\Phi_t = B_t\right] = 0, with$$

$$a_t = E\left[\tilde{V}\,|\Phi_t = B_t\right] = E\left[\tilde{V}\right] + E = \bar{V},$$

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and similarly for the bid price,

$$b_t = E\left[\tilde{V}\,|\Phi_t = S_t\right] = E\left[\tilde{V}\right] - E = \underline{V}.$$

If the next trader is uninformed, market makers' best guess of the value of the security $E\left[\tilde{V}\right] = V^e$ remains. After observing the direction of the trade, market makers consider the price-setting rule explained above with no regret. Notice that this "learning" process only concerns dealers' beliefs, whereas by definition the uninformed does not learn anything from market prices and the insiders already know with perfect foresight. Consequently, the probability of the next trader being informed/uninformed is assumed to be constant through time.

In this context, the ask and bid quotes are equal to,

$$a_t = E\left[\tilde{V}\,|B_t\right] = (1-\alpha)\,E\left[\tilde{V}\right] + \alpha E\left[\tilde{V}\,|B_t\right] = (1-\alpha)\,V^e + \alpha\bar{V}, and$$

$$b_t = E\left[\tilde{V}\,|S_t\right] = (1-\alpha)\,E\left[\tilde{V}\right] + \alpha E\left[\tilde{V}\,|S_t\right] = (1-\alpha)\,V^e + \alpha\underline{V},$$

respectively.

Furthermore,

$$a_t = (1-\alpha)\,V^e + \alpha\left(V^e + E\right),$$
$$\Rightarrow V^e - \alpha V^e + \alpha V^e + \alpha E = V^e + \alpha E.$$

and,

$$b_t = (1-\alpha)\,V^e + \alpha\left(V^e - E\right),$$
$$\Rightarrow V^e - \alpha V^e + \alpha V^e - \alpha E = V^e - \alpha E.$$

The adverse-selection component of the bid-ask spread is equal to,

$$a_t - b_t = \alpha\left(\bar{V} - \underline{V}\right) = V^e + \alpha E - V^e + \alpha E = 2\alpha E. \tag{1.1}$$

Notice that the spread is directly proportional to the probability of informed trading and the insiders' information advantage (i.e., the adverse selection cost). Also, recall that the cost of liquidity per trade for an uninformed trader that seeks immediacy is $\frac{1}{2}$ the bid-ask spread.

Furthermore, the variance of the risky asset is equal to,

$$Var\left(\tilde{V}\right) = \frac{1}{2}\left(\bar{V} - V^e\right)^2 + \frac{1}{2}\left(\underline{V} - V^e\right)^2 = \frac{1}{4}\left(\bar{V} - \underline{V}\right)^2 = \frac{1}{4}4\alpha^2E^2 = \alpha^2E^2, \tag{1.2}$$

where the insider's information advantage or agency cost is equal to,

$$E = \sqrt{\frac{Var\left(\tilde{V}\right)}{\alpha^2}}. \tag{1.3}$$

It follows from (1) and (3) that the bid-ask spread is an increasing function of the variance of the security too. As the market maker observes the incoming buy and sell orders at times $t+1$, she will update her prior about the security's value using Bayes' rule. As time passes, and market makers observe more and more orders their estimate becomes more precise and the spread decreases, in the limit to zero, converging to the hidden true value of the security $V^e \to V^{hidden}$.

Note that in this setting the signed order flow or the net order flow imbalance is a sufficient statistic for the whole story of the past order flow. This implies that the "no-trade event" does not alter the market makers' beliefs.

Assume that $\underline{V} = 0$ and $\bar{V} = 1$. The market makers' conditional belief will be equal to the midpoint between the ask and the bid only if she is trading with uninformed traders i.e., random buying and selling orders with equal probability $\frac{1}{2}$. To the contrary, if market makers are observing more buy/sell orders than sell/buy orders, another buy/sell order will have a smaller impact on the posterior estimate than an additional sell/buy order, and consequently the bid-ask midpoint will shift downwards/upwards. That is, although transaction prices follow a martingale process, quoted bid and ask prices do not, as the increasing number of insiders will increase the serial correlation of the order flow. Note that in this setting markets are semi-strong efficient.

Claim. The spread (1) is the market makers' breakeven spread ignoring the transaction cost component from supplying liquidity to the markets.

Proof. If the next trader is uninformed and a seller, the market maker's profit is,

$$V^e - b.$$

If the next trader is informed and a seller, her profit is,

$$V^e - E - b.$$

Because the market maker does not know with certainty if the next trader is informed or not then her expected profit is,

$$\begin{aligned} &(1-\alpha)(V^e - b) + \alpha(V^e - E - b) \\ &= V^e - b - \alpha V^e + \alpha b + \alpha V^e - \alpha E - \alpha b = V^e - b - \alpha E. \end{aligned}$$

On the other hand, if the next trader is uninformed and a buyer the market maker's profit is,

$$a - V^e.$$

If the next trader is informed and a buyer her profit is,

$$a - (V^e + E).$$

Because the market maker does not know with certainty if the next trader is informed or not then her expected profit is,

$$\begin{aligned} &(1-\alpha)(a - V^e) + \alpha(a - (V^e + E)) \\ &= a - V^e - \alpha a + \alpha V^e + \alpha a - \alpha V^e - \alpha E = a - V^e - \alpha E. \end{aligned}$$

Because the market maker expects the next trader to be equally likely a buyer or a seller, then the breakeven expected profit is,

$$\frac{1}{2}(V^e - b - \alpha E) + \frac{1}{2}(a - V^e - \alpha E) = \frac{1}{2}(a-b) - \alpha E = 0$$

$$\Rightarrow (a-b) = 2\alpha E, \tag{1.4}$$

which is the adverse selection component of the spread. □

Easley and O'Hara (1987) extended this model with order sizes of one (small) and two (large) units. They also introduce the concept of event uncertainty with several trading rounds during the trading day.

References

[Easley and O'Hara (1987) Journal of Financial Economics]
[Glosten and Milgrom (1985) Journal of Financial Economics]