

## INTRODUCTORY MATERIAL FOR PH.D. SEMINAR IN INVESTMENTS

### Part 1. Algebra, Euclidean Spaces, Static Optimization

Reading: Carl Simon and Lawrence Blume, 1985, *Mathematics for Economists*,  
W. W. Norton, NY.

#### 1. LINEAR ALGEBRA (CHAPTERS 6 AND 7)

- 1.1. Linear Systems (Section 6.1).
- 1.2. Investment and Arbitrage (Example 5).
- 1.3. Gaussian and Gauss-Jordan Elimination (Section 7.1).
- 1.4. Elementary Row Operations (Section 7.2).
- 1.5. System with Many or No Solutions (Section 7.3).
- 1.6. Rank (Section 7.4).
- 1.7. Application to Portfolio Theory (Example).
- 1.8. The Linear Implicit Theorem (Section 7.5).

#### 2. MATRIX ALGEBRA (CHAPTER 8)

#### 3. EUCLIDEAN SPACES (CHAPTER 10)

- 3.1. Points and Vectors in Euclidean Space (Section 10.1).
- 3.2. Vectors (Section 10.2).
- 3.3. The Algebra of Vectors (Section 10.3).
- 3.4. Length and Inner Product in  $\mathbb{R}^n$  (Section 10.4).
- 3.5. Lines (Section 10.5).
- 3.6. Planes (Section 10.6).
- 3.7. Applications: Budget Sets in Commodity Space, Probability Simplex, and Investment Model (Examples).

#### 4. SUBSPACES ATTACHED TO A MATRIX (CHAPTER 27)

#### 5. LINEAR INDEPENDENCE (CHAPTER 11)

#### 6. APPLICATIONS OF LINEAR INDEPENDENCE

- 6.1. Geometry of Systems of Equations (Section 28.1).
- 6.2. Portfolio Analysis (Section 28.2).

## 7. STATIC OPTIMIZATION (CHAPTERS 16, 17, 18, 19, AND 30)

- 7.1. **Quadratic Forms (Section 16.1).**
- 7.2. **Definiteness of Quadratic Forms (Section 16.2).**
- 7.3. **Linear Constraints and Bordered Matrices (Section 16.3).**
- 7.4. **Unconstrained Optimization (Section 17.1).**
- 7.5. **First Order Conditions (Section 17.2).**
- 7.6. **Second Order Conditions (Section 17.3).**
- 7.7. **Global Maxima and Minima (Section 17.4).**
- 7.8. **Least Squares Analysis (Example).**
- 7.9. **Examples of Constrained Optimization (Section 18.1).**
- 7.10. **Equality Constraints (Section 18.2).**
- 7.11. **Inequality Constraints (Section 18.3).**
- 7.12. **Mixed Constraints (Section 18.4).**
- 7.13. **Constrained Minimization Problems (Section 18.5).**
- 7.14. **Kuhn-Tucker Formulation (Section 18.6).**
- 7.15. **The Meaning of the Multiplier (Section 19.1).**
- 7.16. **Envelope Theorems (Section 19.2).**
- 7.17. **Weierstrass's and Mean Value Theorems (Section 30.1).**
- 7.18. **Taylor Polynomials on  $\mathbb{R}^1$  (Section 30.2).**
- 7.19. **Taylor Polynomials on  $\mathbb{R}^n$  (Section 30.3).**
- 7.20. **Second Order Optimization Conditions (Section 30.4).**
- 7.21. **Proof of Constrained Optimization (Section 30.5).**

## 8. HOMOGENEOUS AND HOMOTHETIC FUNCTIONS (CHAPTER 20)

## 9. CONCAVE AND QUASICONCAVE FUNCTIONS (CHAPTER 21)

## 10. ECONOMIC APPLICATIONS

- 10.1. **Pareto Optima (Section 22.3).**
- 10.2. **The Fundamental Welfare Theorems (Section 22.4).**

**Part 2. Real Analysis, Markov Chains, Stochastic Dynamic Optimization**

Reading: John Stachurski, 2009, *Economic Dynamics: Theory and Computation*, The MIT Press, Cambridge, MA.

11. INTRODUCTION: THE CONSUMPTION-SAVINGS BENCHMARK DYNAMIC OPTIMIZATION PROBLEM (CHAPTER 1)
12. REAL ANALYSIS (APPENDIX A)
13. OPTIONAL: PROGRAMMING IN PYTHON (CHAPTER 2)
14. INTRODUCTION TO (DISCRETE) DYNAMICS (CHAPTER 4)
  - 14.1. **Finite State Markov Chains (Section 4.2).**
  - 14.2. **Stability of Finite State MCs (Section 4.3).**
15. DYNAMIC PROGRAMMING IN FINITE STATE SPACES (CHAPTER 5)
  - 15.1. **Optimization (Section 5.1).**
  - 15.2. **MCs and SRSs (Section 5.2).**
16. DYNAMIC PROGRAMMING IN INFINITE STATE SPACES (CHAPTER 6)
  - 16.1. **First Steps (Section 6.1).**
  - 16.2. **Stochastic Speculative Price (Section 6.3).**

**Part 3. Probability and Stochastic Processes  
(ADVANCED-OPTIONAL)**

Reading: Patrick Billingsley, 1985, *Probability and Measure*, John Wiley & Sons, NY.

17. PROBABILITY (CHAPTER 1)

- 17.1. **Borel's Normal Number Theorem (Section 1).**
- 17.2. **Probability Measures (Section 2).**
- 17.3. **Simple Random Variables (Section 5).**
- 17.4. **The Law of Large Numbers (Section 6).**
- 17.5. **Markov Chains (Section 8).**

18. RANDOM VARIABLES AND EXPECTED VALUES (CHAPTER 4)

- 18.1. **Random Variables and Distributions (Section 20).**
- 18.2. **Expected Values (Section 21).**

19. CONVERGENCE OF DISTRIBUTIONS (CHAPTER 5)

- 19.1. **The Method of Moments (Section 30).**

20. DERIVATIVES AND CONDITIONAL PROBABILITY (CHAPTER 6)

- 20.1. **Derivatives on the Line (Section 31).**
- 20.2. **The Radon-Nikodym Theorem (Section 32).**
- 20.3. **Conditional Probability (Section 33).**
- 20.4. **Conditional Expectation (Section 34).**
- 20.5. **Martingales (Section 35).**